Formulas to remember

1 Complex numbers.

Let $z = x + iy$ be a complex number.

1. The **conjugate** $\overline{z} = x - iy$.

   The **real part** $Re(z) = x = \frac{z+\overline{z}}{2}$.

   The **imaginary part** $Im(z) = y = \frac{z-\overline{z}}{2i}$.

   The **norm** $|z| = \sqrt{z\overline{z}} = \sqrt{x^2 + y^2}$.

2. The **reciprocal** $\frac{1}{z} = \frac{x-iy}{x^2+y^2}$.

3. The **direction from 0** (for a non zero z) $\frac{z}{|z|}$.

4. The Euler representation. $z = r\exp(i\theta)$ where $r = |z|$ and $\theta = Arg(z) = \arctan(y/z)$. The angle θ can be replaced by any angle in the set $arg(z) = \{Arg(z) + 2n\pi | n \in \mathbb{Z}\}$.

2 Finite series formulas.

1. **Arithmetic series.**

$$a + (a + d) + \cdots + (a + (n-1)d) = \sum_1^n (a + (i-1)d) = \frac{n(2a + (n-1)d)}{2}.$$

2. **Geometric series formula.**

$$a + ar + \cdots ar^{n-1} = \sum_1^n ar^{i-1} = a\frac{1-r^n}{1-r}.$$

3. **Telescoping series formula.** If $g(x) = f(x) - f(x-1)$, then

$$\sum_1^n g(i) = f(n) - f(0).$$

4. **Sums of powers.**

   (a)

$$\sum_1^n 1 = n.$$

(b)

$$\sum_1^n i = \frac{n(n+1)}{2}.$$

(c)

$$\sum_1^n i^2 = \frac{n(n+1)(2n+1)}{6}.$$

(d)

$$\sum_1^n i^3 = \frac{n^2(n+1)^2}{4}.$$

(e)

$$\sum_1^n i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}.$$

3 Power series

1. **Geometric series**

$$\sum_0^\infty ax^i = a + ax + ax^2 + \cdots = \frac{a}{1-x} \text{ where } |x| < 1 \ .$$

2. **Exponential series**

$$\exp(x) = \sum_0^\infty x^i = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots \text{ for all } x \in \Re \ .$$

3. **Basic Trigonometric series**

$$\sin(x) = \sum_0^\infty (-1)^i \frac{x^{2i+1}}{(2i+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \text{ for all } x \in \Re \ .$$

$$\cos(x) = \sum_0^\infty (-1)^i \frac{x^{2i}}{(2i)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \text{ for all } x \in \Re \ .$$

4. **Generalized Binomial series**

$$(1+x)^n = \sum_0^\infty {}_nC_i x^i = \sum_0^\infty \frac{(n)(n-1)\cdots(n-i+1)}{i!} x^i$$

where $|x| < 1$ or n is a non negative integer and x is any complex number.

5. **Log series**

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + (-1)^n \frac{x^{n+1}}{n+1} + \cdots \text{ where } x \text{ is real with } |x| < 1.$$

6. **Arctan series**

$$\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots$$

where x is real with $|x| < 1$.

7. **Arcsine series**

$$\arcsin(x) = x + \frac{1}{2}\frac{x^3}{3} + \frac{(1)(3)}{(2)(4)}\frac{x^5}{5} + \frac{(1)(3)(5)}{(2)(4)(6)}\frac{x^7}{7} + \cdots$$

where x is real with $|x| < 1$.

4 Geometry.

1. The determinant of a 2×2 matrix:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

2. Cramer's Rule: To solve

$$ax + by = e \text{ and } cx + dy = f$$

define:

$$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc, \Delta_x = \begin{vmatrix} e & b \\ f & d \end{vmatrix} = ed - fc \text{ and} \Delta_y = \begin{vmatrix} a & e \\ c & f \end{vmatrix} = af - ec.$$

If $\Delta \neq 0$ then the answer is

$$x = \frac{\Delta_x}{\Delta}, y = \frac{\Delta_y}{\Delta}.$$

If $\Delta = 0$ and at least one of Δ_x, Δ_y is non zero then there is no solution.

If all $\Delta, \Delta_x, \Delta_y$ are zero, then there are infinitely many solutions provided each of the two equations has at least one solution.

3. The distance between two points A and B on the real line is $d(A, B) = |A - B|$.

4. For two points $P_1(a_1, b_1)$ and $P_2(a_2, b_2)$ in the xy-plane: The distance between the points is
$$d(P, Q) = \sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2}.$$

5. The parametric two point form of the line containing the points $P_1(a_1, b_1)$ and $P_2(a_2, b_2)$ is
$$x = a_1 + t(a_2 - a_1), y = b_1 + t(b_2 - b_1).$$

6. The compact parametric two point form of the line is
$$(x, y) = P_1 + t(P_2 - P_1) \text{ or } (x, y) = (1 - t)P_1 + tP_2.$$

7. The midpoint of two points A, B is $\frac{A+B}{2}$.

8. The two point form of the equation through the points $P_1(a_1, b_1), P_2(a_2, b_2)$ is
$$(a_2 - a_1)y - (b_2 - b_1)x = a_2b_1 - a_1b_2.$$

9. The slope of the line through two points $P_1(a_1, b_1), P_2(a_2, b_2)$ is
$$\frac{b_2 - b_1}{a_2 - a_1}$$
where the slope is infinite and the line is vertical if $a_1 = a_2$.

10. The slope intercept form of the line joining $P_1(a_1, b_1), P_2(a_2, b_2)$ is $y = mx + c$ where
$m = \dfrac{b_2 - b_1}{a_2 - a_1}$ is the slope and
$c = \dfrac{a_2b_1 - a_1b_2}{a_2 - a_1}$ is the y-intercept.

11. If $0 \neq p$ is the x intercept and $0 \neq q$ is the y-intercept, then the line is:
$$\frac{x}{p} + \frac{y}{q} = 1.$$

12. A line parallel to $ax + by = c$ is $ax + by = k$ for some k.

13. A line perpendicular to $ax + by = c$ is $bx - ay = k$ for some k.

14. The quadratic function $Q(x) = ax^2 + bx + c$ has extremum value at $x = -\frac{b}{2a}$. The value is $\frac{4ac-b^2}{4a}$. It is a maximum if $a < 0$ and minimum if $a > 0$.

15. The equation $ax^2 + bx + c = 0$ with $a \neq 0$ has solutions:

Quadratic Formula. $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

16. The equation of a circle with center (h, k) and radius r is

$$(x-h)^2 + (y-k)^2 = r^2.$$

17. For the circle

$$x^2 + y^2 + ux + vy = w$$

the center is $(-u/2, -v/2)$ and radius is $\sqrt{w + \frac{u^2}{4} + \frac{v^2}{4}}$.

18. A parametric form of the circle $x^2 + y^2 = r^2$ is:

$$(x, y) = \left(r\frac{1-m^2}{1+m^2}, r\frac{2m}{1+m^2} \right)$$

where m is the parameter.

The trigonometry parameterization for the same circle is: $z = r\exp(i\theta)$ or

$$(x, y) = (r\cos(\theta), r\sin(\theta))$$

with parameter θ.

19. A circle with diameter $P_1(a_1, b_1), P_2(a_2, b_2)$ is:

$$(x-a_1)(x-a_2) + (y-b_1)(y-b_2) = 0.$$

20. The distance from a point (p, q) to the line $ax + by + c = 0$ is:

$$\frac{|ap + bq + c|}{\sqrt{a^2 + b^2}}.$$

The expression $ap + bq + c = 0$ if and only if the point lies on the line.

Moreover, the sign of the expression $ap + bq + c$ changes if the point moves from one side of the line to the other.

5 Trigonometry

1. $$\tan(x) = \frac{\sin(x)}{\cos(x)}, \cot(x) = \frac{\cos(x)}{\sin(x)}, \sec(x) = \frac{1}{\cos(x)}, \csc(x) = \frac{1}{\sin(x)}.$$

2. $$\sin^2(x) + \cos^2(x) = 1, \sec^2(x) = 1 + \tan^2(x), \csc^2(x) = 1 + \cot^2(x).$$

3. $$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y).$$

4.
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y).$$

5.
$$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}.$$

6.
$$\cos(-x) = \cos(x), \sin(-x) = -\sin(x).$$

7.
$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x).$$

8.
$$\sin(2x) = 2\sin(x)\cos(x).$$

9.
$$\cos(\frac{x}{2}) = \pm\sqrt{\frac{1+\cos(x)}{2}}.$$
$$\sin(\frac{x}{2}) = \pm\sqrt{\frac{1-\cos(x)}{2}}.$$

The signs need to be fixed by the position of the locator point $P(\frac{t}{2})$.

10.

Sine Law. $$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

11.

Cosine Law. $$a^2 = b^2 + c^2 - 2bc\cos(A)$$

6 Derivative Formulas

1. If c is a constant, then $D_x(c) = 0$. Moreover, $D_x(cy) = cD_x(y)$.
2. **Sum Rule.**
$$D_x(y+z) = D_x(y) + D_x(z).$$
3. **Power Rule.** For any real n,
$$D_x(y^n) = ny^{n-1}D_x(y).$$
As a special case:
$$D_x(x^n) = nx^{n-1}.$$

4. **Product Rule.**
$$D_x(yz) = D_x(y)z + yD_x(z).$$

5. **Quotient Rule.**
$$D_x\left(\frac{y}{z}\right) = \frac{D_x(y)z - yD_x(z)}{z^2}.$$

6. **Chain Rule.**
$$D_x(f(g(x)) = f'(g(x))g'(x).$$

7. **Linear approximation formula:** Linear approximation to $f(x)$ at $x = a$ is:
$$L(x) = f(a) + f'(x)(x - a).$$

8. **Newton's Method** for finding a root.
   - Start with some convenient guess x_0. This is the current guess.
   - If x_n is the current guess then the new current guess shall be
   $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$
   - Stop when successive current guesses are close to each other (within the set accuracy level) or when the value of the function is sufficiently close to zero (within the set accuracy).