

INTEGRALS

1. Common Integrals

Indefinite Integral

Method of substitution

$$\int f(g(x))g'(x)dx = \int f(u)du$$

Integration by parts

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$$

Integrals of Rational and Irrational Functions

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int c dx = cx + C$$

$$\int x dx = \frac{x^2}{2} + C$$

$$\int x^2 dx = \frac{x^3}{3} + C$$

$$\int \frac{1}{x^2} dx = -\frac{1}{x} + C$$

$$\int \sqrt{x} dx = \frac{2x\sqrt{x}}{3} + C$$

$$\int \frac{1}{1+x^2} dx = \arctan x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$$

Integrals of Trigonometric Functions

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \tan x dx = \ln|\sec x| + C$$

$$\int \sec x dx = \ln|\tan x + \sec x| + C$$

$$\int \sin^2 x dx = \frac{1}{2}(x - \sin x \cos x) + C$$

$$\int \cos^2 x dx = \frac{1}{2}(x + \sin x \cos x) + C$$

$$\int \tan^2 x dx = \tan x - x + C$$

$$\int \sec^2 x dx = \tan x + C$$

Integrals of Exponential and Logarithmic Functions

$$\int \ln x dx = x \ln x - x + C$$

$$\int x^n \ln x dx = \frac{x^{n+1}}{n+1} \ln x - \frac{x^{n+1}}{(n+1)^2} + C$$

$$\int e^x dx = e^x + C$$

$$\int b^x dx = \frac{b^x}{\ln b} + C$$

$$\int \sinh x dx = \cosh x + C$$

$$\int \cosh x dx = \sinh x + C$$

2. Integrals of Rational Functions

Integrals involving $ax + b$

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} \quad (\text{for } n \neq -1)$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b|$$

$$\int x(ax+b)^n dx = \frac{a(n+1)x-b}{a^2(n+1)(n+2)} (ax+b)^{n+1} \quad (\text{for } n \neq -1, n \neq -2)$$

$$\int \frac{x}{ax+b} dx = \frac{x}{a} - \frac{b}{a^2} \ln|ax+b|$$

$$\int \frac{x}{(ax+b)^2} dx = \frac{b}{a^2(ax+b)} + \frac{1}{a^2} \ln|ax+b|$$

$$\int \frac{x}{(ax+b)^n} dx = \frac{a(1-n)x-b}{a^2(n-1)(n-2)(ax+b)^{n-1}} \quad (\text{for } n \neq -1, n \neq -2)$$

$$\int \frac{x^2}{ax+b} dx = \frac{1}{a^3} \left(\frac{(ax+b)^2}{2} - 2b(ax+b) + b^2 \ln|ax+b| \right)$$

$$\int \frac{x^2}{(ax+b)^2} dx = \frac{1}{a^3} \left(ax+b - 2b \ln|ax+b| - \frac{b^2}{ax+b} \right)$$

$$\int \frac{x^2}{(ax+b)^3} dx = \frac{1}{a^3} \left(\ln|ax+b| + \frac{2b}{ax+b} - \frac{b^2}{2(ax+b)^2} \right)$$

$$\int \frac{x^2}{(ax+b)^n} dx = \frac{1}{a^3} \left(-\frac{(ax+b)^{3-n}}{n-3} + \frac{2b(a+b)^{2-n}}{n-2} - \frac{b^2(ax+b)^{1-n}}{n-1} \right) \quad (\text{for } n \neq 1, 2, 3)$$

$$\int \frac{1}{x(ax+b)} dx = -\frac{1}{b} \ln \left| \frac{ax+b}{x} \right|$$

$$\int \frac{1}{x^2(ax+b)} dx = -\frac{1}{bx} + \frac{a}{b^2} \ln \left| \frac{ax+b}{x} \right|$$

$$\int \frac{1}{x^2(ax+b)^2} dx = -a \left(\frac{1}{b^2(a+xb)} + \frac{1}{ab^2x} - \frac{2}{b^3} \ln \left| \frac{ax+b}{x} \right| \right)$$

Integrals involving $ax^2 + bx + c$

$$\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \operatorname{arctg} \frac{x}{a}$$

$$\int \frac{1}{x^2-a^2} dx = \begin{cases} \frac{1}{2a} \ln \frac{a-x}{a+x} & \text{for } |x| < |a| \\ \frac{1}{2a} \ln \frac{x-a}{x+a} & \text{for } |x| > |a| \end{cases}$$

$$\int \frac{1}{ax^2 + bx + c} dx = \begin{cases} \frac{2}{\sqrt{4ac - b^2}} \arctan \frac{2ax + b}{\sqrt{4ac - b^2}} & \text{for } 4ac - b^2 > 0 \\ \frac{2}{\sqrt{b^2 - 4ac}} \ln \left| \frac{2ax + b - \sqrt{b^2 - 4ac}}{2ax + b + \sqrt{b^2 - 4ac}} \right| & \text{for } 4ac - b^2 < 0 \\ -\frac{2}{2ax + b} & \text{for } 4ac - b^2 = 0 \end{cases}$$

$$\int \frac{x}{ax^2 + bx + c} dx = \frac{1}{2a} \ln |ax^2 + bx + c| - \frac{b}{2a} \int \frac{dx}{ax^2 + bx + c}$$

$$\int \frac{mx + n}{ax^2 + bx + c} dx = \begin{cases} \frac{m}{2a} \ln |ax^2 + bx + c| + \frac{2an - bm}{a\sqrt{4ac - b^2}} \arctan \frac{2ax + b}{\sqrt{4ac - b^2}} & \text{for } 4ac - b^2 > 0 \\ \frac{m}{2a} \ln |ax^2 + bx + c| + \frac{2an - bm}{a\sqrt{b^2 - 4ac}} \operatorname{arctanh} \frac{2ax + b}{\sqrt{b^2 - 4ac}} & \text{for } 4ac - b^2 < 0 \\ \frac{m}{2a} \ln |ax^2 + bx + c| - \frac{2an - bm}{a(2ax + b)} & \text{for } 4ac - b^2 = 0 \end{cases}$$

$$\int \frac{1}{(ax^2 + bx + c)^n} dx = \frac{2ax + b}{(n-1)(4ac - b^2)(ax^2 + bx + c)^{n-1}} + \frac{(2n-3)2a}{(n-1)(4ac - b^2)} \int \frac{1}{(ax^2 + bx + c)^{n-1}} dx$$

$$\int \frac{1}{x(ax^2 + bx + c)} dx = \frac{1}{2c} \ln \left| \frac{x^2}{ax^2 + bx + c} \right| - \frac{b}{2c} \int \frac{1}{ax^2 + bx + c} dx$$

3. Integrals of Exponential Functions

$$\int xe^{cx} dx = \frac{e^{cx}}{c^2} (cx - 1)$$

$$\int x^2 e^{cx} dx = e^{cx} \left(\frac{x^2}{c} - \frac{2x}{c^2} + \frac{2}{c^3} \right)$$

$$\int x^n e^{cx} dx = \frac{1}{c} x^n e^{cx} - \frac{n}{c} \int x^{n-1} e^{cx} dx$$

$$\int \frac{e^{cx}}{x} dx = \ln |x| + \sum_{i=1}^{\infty} \frac{(cx)^i}{i \cdot i!}$$

$$\int e^{cx} \ln x dx = \frac{1}{c} e^{cx} \ln |x| + E_i(cx)$$

$$\int e^{cx} \sin bxdx = \frac{e^{cx}}{c^2 + b^2} (c \sin bx - b \cos bx)$$

$$\int e^{cx} \cos bxdx = \frac{e^{cx}}{c^2 + b^2} (c \cos bx + b \sin bx)$$

$$\int e^{cx} \sin^n x dx = \frac{e^{cx} \sin^{n-1} x}{c^2 + n^2} (c \sin x - n \cos bx) + \frac{n(n-1)}{c^2 + n^2} \int e^{cx} \sin^{n-2} dx$$

4. Integrals of Logarithmic Functions

$$\int \ln cx dx = x \ln cx - x$$

$$\int \ln(ax + b) dx = x \ln(ax + b) - x + \frac{b}{a} \ln(ax + b)$$

$$\int (\ln x)^2 dx = x(\ln x)^2 - 2x \ln x + 2x$$

$$\int (\ln cx)^n dx = x(\ln cx)^n - n \int (\ln cx)^{n-1} dx$$

$$\int \frac{dx}{\ln x} = \ln |\ln x| + \ln x + \sum_{n=2}^{\infty} \frac{(\ln x)^n}{n \cdot n!}$$

$$\int \frac{dx}{(\ln x)^n} = -\frac{x}{(n-1)(\ln x)^{n-1}} + \frac{1}{n-1} \int \frac{dx}{(\ln x)^{n-1}} \quad (\text{for } n \neq 1)$$

$$\int x^m \ln x dx = x^{m+1} \left(\frac{\ln x}{m+1} - \frac{1}{(m+1)^2} \right) \quad (\text{for } m \neq -1)$$

$$\int x^m (\ln x)^n dx = \frac{x^{m+1} (\ln x)^n}{m+1} - \frac{n}{m+1} \int x^m (\ln x)^{n-1} dx \quad (\text{for } m \neq -1)$$

$$\int \frac{(\ln x)^n}{x} dx = \frac{(\ln x)^{n+1}}{n+1} \quad (\text{for } n \neq -1)$$

$$\int \frac{\ln x^n}{x} dx = \frac{(\ln x^n)^2}{2n} \quad (\text{for } n \neq 0)$$

$$\int \frac{\ln x}{x^m} dx = -\frac{\ln x}{(m-1)x^{m-1}} - \frac{1}{(m-1)^2 x^{m-1}} \quad (\text{for } m \neq 1)$$

$$\int \frac{(\ln x)^n}{x^m} dx = -\frac{(\ln x)^n}{(m-1)x^{m-1}} + \frac{n}{m-1} \int \frac{(\ln x)^{n-1}}{x^m} dx \quad (\text{for } m \neq 1)$$

$$\int \frac{dx}{x \ln x} = \ln |\ln x|$$

$$\int \frac{dx}{x^n \ln x} = \ln |\ln x| + \sum_{i=1}^{\infty} (-1)^i \frac{(n-1)^i (\ln x)^i}{i \cdot i!}$$

$$\int \frac{dx}{x(\ln x)^n} = -\frac{1}{(n-1)(\ln x)^{n-1}} \quad (\text{for } n \neq 1)$$

$$\int \ln(x^2 + a^2) dx = x \ln(x^2 + a^2) - 2x + 2a \tan^{-1} \frac{x}{a}$$

$$\int \sin(\ln x) dx = \frac{x}{2} (\sin(\ln x) - \cos(\ln x))$$

$$\int \cos(\ln x) dx = \frac{x}{2} (\sin(\ln x) + \cos(\ln x))$$

5. Integrals of Trig. Functions

$$\int \sin x dx = -\cos x$$

$$\int \cos x dx = \sin x$$

$$\int \sin^2 x dx = \frac{x}{2} - \frac{1}{4} \sin 2x$$

$$\int \cos^2 x dx = \frac{x}{2} + \frac{1}{4} \sin 2x$$

$$\int \sin^3 x dx = \frac{1}{3} \cos^3 x - \cos x$$

$$\int \cos^3 x dx = \sin x - \frac{1}{3} \sin^3 x$$

$$\int \frac{dx}{\sin x} = \ln \left| \tan \frac{x}{2} \right|$$

$$\int \frac{dx}{\cos x} = \ln \left| \tan \left(\frac{x}{2} + \frac{\pi}{4} \right) \right|$$

$$\int \frac{dx}{\sin^2 x} = -\cot x$$

$$\int \frac{dx}{\cos^2 x} = \tan x$$

$$\int \frac{dx}{\sin^3 x} = -\frac{\cos x}{2 \sin^2 x} + \frac{1}{2} \ln \left| \tan \frac{x}{2} \right|$$

$$\int \frac{dx}{\cos^3 x} = \frac{\sin x}{2 \cos^2 x} + \frac{1}{2} \ln \left| \tan \left(\frac{x}{2} + \frac{\pi}{4} \right) \right|$$

$$\int \sin x \cos x dx = -\frac{1}{4} \cos 2x$$

$$\int \sin^2 x \cos x dx = \frac{1}{3} \sin^3 x$$

$$\int \sin x \cos^2 x dx = -\frac{1}{3} \cos^3 x$$

$$\int \sin^2 x \cos^2 x dx = \frac{x}{8} - \frac{1}{32} \sin 4x$$

$$\int \tan x dx = -\ln |\cos x|$$

$$\int \frac{\sin x}{\cos^2 x} dx = \frac{1}{\cos x}$$

$$\int \frac{\sin^2 x}{\cos x} dx = \ln \left| \tan \left(\frac{x}{2} + \frac{\pi}{4} \right) \right| - \sin x$$

$$\int \tan^2 x dx = \tan x - x$$

$$\int \cot x dx = \ln |\sin x|$$

$$\int \frac{\cos x}{\sin^2 x} dx = -\frac{1}{\sin x}$$

$$\int \frac{\cos^2 x}{\sin x} dx = \ln \left| \tan \frac{x}{2} \right| + \cos x$$

$$\int \cot^2 x dx = -\cot x - x$$

$$\int \frac{dx}{\sin x \cos x} = \ln |\tan x|$$

$$\int \frac{dx}{\sin^2 x \cos x} = -\frac{1}{\sin x} + \ln \left| \tan \left(\frac{x}{2} + \frac{\pi}{4} \right) \right|$$

$$\int \frac{dx}{\sin x \cos^2 x} = \frac{1}{\cos x} + \ln \left| \tan \frac{x}{2} \right|$$

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \tan x - \cot x$$

$$\int \sin mx \sin nx dx = -\frac{\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)} \quad m^2 \neq n^2$$

$$\int \sin mx \cos nx dx = -\frac{\cos(m+n)x}{2(m+n)} - \frac{\cos(m-n)x}{2(m-n)} \quad m^2 \neq n^2$$

$$\int \cos mx \cos nx dx = \frac{\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)} \quad m^2 \neq n^2$$

$$\int \sin x \cos^n x dx = -\frac{\cos^{n+1} x}{n+1}$$

$$\int \sin^n x \cos x dx = \frac{\sin^{n+1} x}{n+1}$$

$$\int \arcsin x dx = x \arcsin x + \sqrt{1-x^2}$$

$$\int \arccos x dx = x \arccos x - \sqrt{1-x^2}$$

$$\int \arctan x dx = x \arctan x - \frac{1}{2} \ln(x^2 + 1)$$

$$\int \operatorname{arc cot} x dx = x \operatorname{arc cot} x + \frac{1}{2} \ln(x^2 + 1)$$